APPLICATION OF CIPM 2007 FORMULA AND ITS INFLUENCE IN DETERMINING UNCERTAINTY DUE TO VARIATION OF AIR DENSITY

APLIKASI PERSAMAAN CIPM 2007 DAN PENGARUHNYA PADA PENENTUAN KETIDAKPASTIAN AKIBAT VARIASI DENSITAS UDARA

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ABSTRACT

Air density measurement is an important parameter in standard mass calibration. The value is calculated from four measured components, i.e. temperature, relative humidity, mole fraction of carbon dioxide, and air pressure of environment where the calibration takes place. Empirical formula to calculate air density is recommended by *Comité International des Poids et Measures* (CIPM) in 1981/91, then updated in 2007. In this paper, there is explanation about the differences between CIPM 1981/91 formula and CIPM 2007, their applications to air density calculation and the effects into uncertainty budget, and uncertainty contribution from air density variation to the standard mass calibration. The data are taken from the environment condition in mass laboratory of RCM LIPI. From the calculation, the average air density obtained from CIPM 1981/91 formula is 0.0001 kgm^{-3} smaller than that obtained from CIPM 2007 formula. Air density variation in the mass laboratory is calculated as 0.00572 kgm^{-3} using CIPM 1981/91 formula and 0.00566 kgm^{-3} using CIPM 2007 formula. Uncertainty contribution to standard mass calibration for both formulas are relatively the same, about 0.031 kgm^{-3}, with the sensitivity coefficient 3.0 x 10^{-6} m^3, making it the third largest contributor after mass standard and instability into the uncertainty budget of mass calibration.

Keywords: air density, standard mass, CIPM 2007, CIPM 1981/91

ABSTRAK

Densitas udara adalah parameter yang penting untuk diperhitungkan pada kalibrasi massa standar. Nilainya ditentukan oleh pengukuran empat besaran, yaitu temperatur, kelembapan relatif, fraksi mol karbondioksida, dan tekanan udara pada ruang tempat kalibrasi dilakukan. Formula empiris yang digunakan untuk menghitung direkomendasikan oleh CIPM pada tahun 1981/91, yang kemudian diperbaharui pada tahun 2007. Pada tulisan ini akan dijelaskan perbedaan formula CIPM 1981/91 dan CIPM 2007, aplikasinya terhadap perhitungan densitas udara, pengaruhnya terhadap perhitungan ketidakpastian, dan kontribusi ketidakpastian densitas udara terhadap kalibrasi massa standar. Data yang digunakan adalah kondisi ruangan laboratorium massa Puslit Metrologi LIPI yang dicatat selama sebulan. Dari hasil perhitungan, didapat rata-rata densitas udara dengan formula CIPM 1981/91 sebesar 0,0001 kgm^{-3} *lebih kecil daripada yang dihitung dengan formula CIPM 2007. Variasi densitas udara terhitung sebesar 0,00572* kgm^{-3} *dari formula CIPM 1981/91 dan 0,00566 dari formula CIPM 2007. Kontribusi ketidakpastian pada kalibrasi massa standar untuk kedua formula tidak jauh berbeda, sekitar 0,031* kgm^{-3} *dengan koefisien sensitivitas 3,0 x* 10^{-6} m^3*, menjadikannya sebagai kontributor ketidakpastian ketiga terbesar setelah massa standard an ketidakstabilan pada budget ketidakpastian kalibrasi massa standar.*

Kata kunci: *densitas udara, massa standar, CIPM 2007, CIPM 1981/91*

A. INTRODUCTION

In the process of a standard mass calibration in air, air density around the balance has a significant contribution to the uncertainty budget. The impact will be highly visible for standard mass calibration with very different volumes. For calibration of weights class E2 and E1, air density (buoyancy correction) calculation is needed to get the right conventional mass value (Jones & Scoonover, 2002).

Air density is a parameter that cannot be measured directly. It needs to be calculated using a formula recommended by CIPM in 1981. The formula has been always updated following a result of the latest research, once in 1991 a small update was added, so the formula was called CIPM 1981/91 (Giacomo, 1982; Davis, 1992). The latest update was released in 2007 thus called CIPM 2007 (Picard, Davis, Glaser, & Fujii, 2008). Air density is calculated from four measured parameters, i.e. temperature, relative humidity, air pressure, and mole fraction of carbon dioxide in air. There are some assumptions and approaches made in the formula to make the calculation easier without reducing the validity of the calculation result.

This paper discusses the differences between air density calculation using CIPM 1981/91 formula and CIPM 2007 formula. Air density is calculated from the measured temperature, air pressure, and relative humidity in mass laboratory of Research Center for Metrology (RCM LIPI). The values are recorded periodically for one month. The differences in the result will be calculated and analyzed to the uncertainty budget

Another factor contributed to air density measurement is the mole fraction of carbon dioxide. Because the lack of measurement instrument for the mole fraction of carbondioxide, hence the value is assumed as in the paper (Picard et al., 2008). The data of standard mass calibration are taken in the same laboratory. The result of weight calibration is calculated and analyzed to see the impact of buoyancy correction to the uncertainty budget.

B. BASIC THEORY

The equation used in CIPM 2007 formula is basically the same as the one used in CIPM 1981/91,

$$\rho = \frac{pM_a}{ZRT}\left[1 - x_v\left(1 - \frac{M_v}{M_a}\right)\right] \tag{1}$$

where,

ρ is air density (kgm^{-3})

p is pressure (Pa)

Z is compressibility factor

R is molar gas constant ($Jmol^{-1}\ K^{-1}$)

T is air temperature (K)

x_v is mole fraction of water vapour

M_a is molar mass of dry air ($g\ mol^{-1}$)

M_v is molar mass of water ($g\ mol^{-1}$)

The difference lies in some variables that use the value from the result of newer researches and approaches.

1. Molar gas constant (*R*)

In CIPM 1981/91 formula, the value of R recommended by CODATA 1986 (Cohen & Taylor, 1986) is 8,314510 ($1 \pm 8{,}4 \times 10^{-6}$) $Jmol^{-1}\ K^{-1}$. For CIPM 2007 the value is updated by CODATA 2006 (Mohr &Taylor, 2005) to 8,314472(15) $Jmol^{-1}\ K^{-1}$

2. Molar mass of dry air (M_a)

The value for molar mass of dry air is changed because the discrepancy of the Argon mole fraction (x_{Ar}) in air. In CIPM 1981/91, x_{Ar} value is 9,17 mmol mol-1, based on calculation by Chacket, Paneth, & Wilson (1949). For CIPM 2007 the value of x_{Ar} is changed to 9,332(3) mmol mol-1, based on the recent calculations by KRISS and LNE (Park, Kim, Lee, Esler, Davis, Wielgosz, 2004; Sutour, Stumpf, Kosinski, Surget, Hervouët, Yardin, ... & Gosset, 2007).

The discrepancy thus changes the value of nitrogen mole fraction in atmosphere. Here are the differences between composition of air used in both CIPM formulas shown in Table 1.

In this research, mole fraction of CO_2 was not measured, so it is assumed to have the value of 400 $\mu mol\ mol^{-1}$. Both the CIPM formulas calculate the molar mass of dry air (M_a) using the same equation, but the discrepancy in the composition of mole fractions made difference in the value of M_a obtained. M_a is calculated by equation [2].

$$M_a = \frac{\sum M_i x_i}{\sum x_i} \tag{2}$$

The value of M_a for CIPM 1981/91 formula is $28{,}9635 \times 10^{-3}$ kg mol^{-1} and for CIPM 2007 the value is $28{,}96546 \times 10^{-3}$ kg mol^{-1}.

3. Mole fraction of water vapour (x_v)

Calculation for x_v is unchanged with the same constant variables. The equation is:

$$x_v = hf(p,t) \cdot \frac{p_{sv}(t)}{p} \tag{3}$$

Table 1. Composition of Dry Air

	CIPM 1981/91			CIPM 2007		
Constituent	Molar mass (M_i)	Mole fraction (x_i)	($M_i x_i$)	Molar mass (M_i)	Mole fraction (x_i)	($M_i x_i$)
N	28.0134	0.78101	21.878746	28.0134	0.780848	21.874207
O	31.9988	0.20939	6.700229	31.9988	0.20939	6.700229
Ar	39.948	0.00917	0.366323	39.9480	0.009332	0.372795
CO_2	44.01	0.000400	0.017604	44.01	0.000400	0.017604
Ne	20.18	0.000018	0.000367	20.18	0.000018	0.000367
He	4.0	5.2×10^{-6}	0.000021	4.0	5.2×10^{-6}	0.000021
CH_4	16.0	1.5×10^{-6}	0.000024	16.0	1.5×10^{-6}	0.000024
Kr	83.8	1.1×10^{-6}	0.000092	83.8	1.1×10^{-6}	0.000092
H_2	2	5.0×10^{-7}	0.000001	2	5.0×10^{-7}	0.000001
N_2O	44	3.0×10^{-7}	0.000013	44	3.0×10^{-7}	0.000013
CO_2	28	2.0×10^{-7}	0.000006	28	2.0×10^{-7}	0.000006
Xe	131	1.0×10^{-7}	0.000013	131	1.0×10^{-7}	0.000013

$$f = \alpha + \beta p + \gamma t^2 \quad (4)$$

$$p_{sv} = 1Pa\ x\ exp\left(AT^2 + BT + C + D/T\right) \quad (5)$$

where:
h is relative humidity (%)
f is enhancement factor
α is a constant value 1,00062
β is $3,14 \times 10^{-8}$ Pa^{-1}
γ is $5,6 \times 10^{-7}$ K^{-2}
t is temperature (°C)
p_{sv} is vapour pressure of saturated air
A is $1,2378847 \times 10^{-5}$ K^{-2}
B is $-1,9121316 \times 10^{-2}$ K^{-1}
C is 33,93711047
D is $-6,3431645 \times 10^{3}$ K
T is Temperatur (K)

4. Compressibility factor (Z)

For both CIPM formulas, compressibility factor (Z) is calculated by the same equation below.

$$Z = 1 - \frac{P}{T}[a_0 + a_1 t + a_2 t^2 + \left(b_0 + b_1 t\right) x_v + \left(c_0 + c_1 t\right) x_v{}^2] + \frac{p^2}{T^2}\left(d + e x_v{}^2\right) \quad (6)$$

where:
a_0 is $1,58123 \times 10^{-6}$ KPa^{-1}
a_1 is $-2,9331 \times 10^{-8}$ Pa^{-1}
a_2 is $1,1043 \times 10^{-10}$ K^{-1} Pa^{-1}
b_0 is $5,707 \times 10^{-6}$ KPa^{-1}
b_1 is $-2,051 \times 10^{-8}$ Pa^{-1}
c_0 is $1,9898 \times 10^{-4}$ KPa^{-1}
c_1 is $-2,376 \times 10^{-6}$ Pa^{-1}
d is $1,83 \times 10^{-11}$ K^2 Pa^{-2}
e is $-0,765 \times 10^{-8}$ K^2 Pa^{-2}

5. Uncertainty Calculation of Air Density

Uncertainty calculation of air density for both CIPM formulas is calculated by the same equation below.

$$u^2 \rho_a = u_f{}^2 + \left(\frac{\partial \rho_a}{\partial_p} u_p\right)^2 + \left(\frac{\partial \rho_a}{\partial_t} u_t\right)^2 + \left(\frac{\partial \rho_a}{\partial h} u_h\right)^2 + \left(\frac{\partial \rho_a}{\partial t_d} u_{t_d}\right)^2 + \left(\frac{\partial \rho_a}{\partial xco_2} u_{xco_2}\right)^2 + rms\left(\rho_a - 1.2\right)^2 \quad (7)$$

Where the constant value was described in Table 2 below.

The variables are basically the same, except for the uncertainty component from the formulas, which CIPM 2007 is smaller than CIPM 1981/91. Uncertainty components from humidity and dew-point temperature can be included either one. In this research, humidity

Table 2. Components of Uncertainty

Variables	Uncertainty Components	CIPM 1981/91	CIPM 2007
$u^2\rho_a$	Air density		
u_f	Formula	$+2.5\times10^{-5}$	$+2.2\times10^{-5}$
$\frac{u_p(\rho_a)}{\rho_a}$	Air pressure	$+1\times10^{-5}Pa^{-1}\cdot u(p)$	
$\frac{u_T(\rho_a)}{\rho_a}$	Temperature	$-4\times10^{-3}K^{-1}\cdot u(T)$	
$\frac{u_h(\rho_a)}{\rho_a}$	Humidity	$-9\times10^{-3}\cdot u(h)$	
$\frac{u_{t_d}(\rho_a)}{\rho_a}$	Dew-Point Temperature	$-3\times10^{-4}K^{-1}\cdot u(t_d)$	
$\frac{u_{xco_2}(\rho_a)}{\rho_a}$	Carbon Mole Fraction	$+0.4\cdot u(xco_2)$	

component is added because relative humidity is measured.

Uncertainty of air density is also known as buoyancy correction. Sensitivity coefficient is derived from this equation below.

$$b=(\rho_{udara}-1.2)\left(\frac{1}{\rho_t}-\frac{1}{\rho_s}\right)M \quad (8)$$

$$c_b=\frac{\partial b}{\partial\rho_{udara}}=\left(\frac{1}{\rho_t}-\frac{1}{\rho_s}\right)M \quad (9)$$

From the table in OIML R111-1 document (Organisation Internationale de Metrologie Legale, 2004), density of the weight that will be calibrated and the standard weight are assumed as displayed in Table 3.

Table 3. Density of Weights

Density (kgm-3)	
E1	8000
E2	7810

C. MEASUREMENT METHOD

The data are recorded in Mass Laboratory of Research Center for Metrology LIPI (RCM LIPI) for 1 month, between 1st February 2016 to 29th February 2016. The measured values are temperature, air pressure, and relative humidity. The data are recorded three times a day at 9 a.m., 11 a.m., and 3 p.m.

Measurement instruments used are thermohygrometer FLUKE 1620A “DEWK” for temperature and humidity (Figure1), and Pressure Wallace & Tiernan for air pressure (Figure 2). Both instruments are traceable to RCM LIPI as the National Metrology Institute.

Figure 1. Temperature and Humidity Sensor

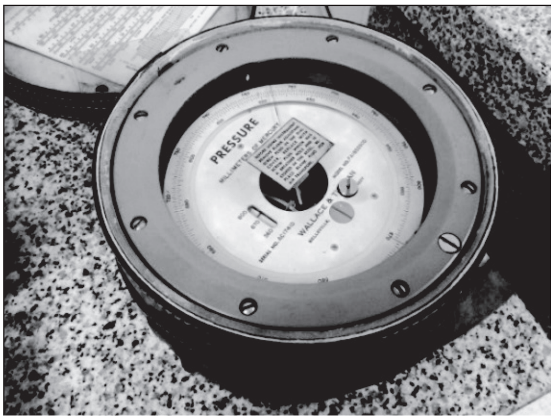

Figure 2. Air Pressure Sensor

Figure 3. Mass Calibration System

Mass calibration data is taken from the calibration of 1000 gram weight of class E2 Mettler Toledo with serial number 01-158850 which is calibrated by weight E1 with serial number 01/159351. Weighing is done by a comparator mass AX1005 which has been traceable to RCM LIPI. The calibration method is direct comparison between the weight of mass standard and the weight under test.

The calibration system is as seen in Figure 3. Inside the AX1005 mass compator is the mass standard being weighed. The weight under test is placed beside the comparator. Both of weights are weighed alternately using the scheme S-T-T-S (Standard-Test-Test-Standard).

D. RESULTS AND DISCUSSION

The three measured values are temperature in Celsius, relative humidity in percent, and air pressure in milibar. The data are averaged and then calculated by CIPM 1981/91 and CIPM 2007 formulas to find the air density.

From Table 4 we know that the discrepancy between the average air density calculated by CIPM 1981/91 and CIPM 2007 can be seen in the fourth digit behind the comma. The difference value is 0.0001 kgm^{-3} with the result of CIPM 2007 is bigger than that of CIPM 1981/91.

For the calculation of variation (root mean square), the difference can be seen in the fourth digit behind the comma. Where the result from CIPM 1981/91 is 0.00006 kgm^{-3} bigger than that from CIPM 2007.

Environment requirement for class E2 weight calibration is temperature variation around ± 0.7°C per hour with a maximum of ± 1°C per twelve hours, humidity variation around 40% to 60% with a maximum of ± 10% per four hours, and air density variation around 1.2 kgm^{-3} by more than 10%. Air density variation 0.00572 kgm^{-3}or 0.47% from 1.2 kgm^{-3} for the formula CIPM 1981/91. For CIPM 2007 air density variation is 0.00566 kgm^{-3} or 0.47% from 1.2 kgm^{-3}. Those values are considered acceptable, means that the environment condition of the calibration laboratory is suitable for calibration.

Table 4. Average Air Density and Variation

	$\rho_{CIPM1981/91}$ (kgm^{-3})	$\rho_{CIPM2007}$ (kgm^{-3})
Average ρ_a	1.1963	1.1964
Variation	0.00572	0.00566

Table 5. Uncertainty Budget of Air Density

Variables	Unit	u_i	c_i	CIPM 1981/91		CIPM 2007	
				$u_i \cdot c_i$	$(u_i \cdot c_i)^2$	$u_i \cdot c_i$	$(u_i \cdot c_i)^2$
u_f	-	-	-	2.50×10^{-5}	6.25×10^{-10}	2.20×10^{-5}	4.84×10^{-10}
u_p	Pascal	14.43	10^{-5}	1.44×10^{-4}	2.08×10^{-8}	1.44×10^{-4}	2.08×10^{-8}
u_T	Kelvin	0.57	–0.004	–2.28×10^{-3}	5.20×10^{-6}	–2.28×10^{-3}	5.20×10^{-6}
u_h	-	3.40	–0.009	–0.0306	9.36×10^{-4}	–0.0306	9.36×10^{-4}
u_{xco_2}	-	8.3×10^{-6}	0.4	3.32×10^{-6}	1.10×10^{-11}	3.32×10^{-6}	1.10×10^{-11}
rms	kg/m^3	-	-	5.72×10^{-3}	3.27×10^{-5}	5.66×10^{-3}	3.21×10^{-5}
					9.74×10^{-4}		9.74×10^{-4}
				$u_{\rho a}$	0.03121		0.03120

Table 6. Uncertainty Budget for 1000 g Weight Class E2

Komponen	Unit	Dist	U	div	u_i	c_i	v_i	$u_i c_i$	$(u_i c_i)^2$	$\frac{(u_i \cdot c_i)^4}{v_i}$
Mass Standard	mg	Normal	0.06	2	0.03	1	58	0.03	0.0009	1.4×10^{-8}
Repeatability	mg	Normal	4.6×10^{-5}	$\sqrt{3}$	2.6×10^{-5}	1	3	2.6×10^{-5}	7.0×10^{-10}	1.6×10^{-19}
Resolution	mg	Rect	$5.0\ 10^{-6}$	$\sqrt{3}$	4.1×10^{-6}	1	10^9	4.1×10^{-6}	1.7×10^{-11}	2.8×10^{-31}
Instability	mg	Rect	0.07	$\sqrt{3}$	0.04	1	10^9	0.04	0.0016	2.5×10^{-15}
Buoyancy	mg	Normal	31	1	31	3.0×10^{-6}	59	9.5×10^{-5}	9.0×10^{-9}	1.4×10^{-30}
								sum	0.0025	1.4×10^{-8}
								u_{comb}	0.05	6.3×10^{-6}
								v eff	447	
								coverage factor	1.99	
								$u_{expanded}$	0.10	

As seen on the Table 5, the differences between uncertainty budget for CIPM 198/91 and CIPM 2007 formula lie on the value of variation (root mean square) and uncertainty of the formulas. The result of uncertainty budget showed that the difference is about 0.00001 with the result from CIPM 1981/91 is bigger than that from CIPM 2007.

The small difference is insignificant because for uncertainty calculation only two important numbers are taken.This result is consistent with the conclusion in the paper by A Picard et al. (2008). It can be said that the uncertainty value for both formulas is 0.031kgm^{-3}.

The highest component contributing to the uncertainty budget is relative humidity, then air density variation inside the laboratory, and temperature. Uncertainty from relative humidity and temperature are caused by the large multiplier factor of sensitivity coefficient and limitation of the measurement instrument. Air density variation inside the laboratory is caused by the fluctuating temperature, relative humidity, and air pressure.

Uncertainty component from mole fraction of carbon dioxide comes the least. The impact is very small; it can be neglected. That is why the value of mole fraction of carbon dioxide can be assumed.

From Table 6 we know that in the uncertainty budget for weight calibration, buoyancy correction or air density takes place as the third highest contributor, after instability and mass standard component.

E. CONCLUSION

There are several differences between CIPM 1981/91 and CIPM 2007 formulas: molar gas constant (R), molar mass of dry air (M_a), and the uncertainty from the formula. The differences make a slight discrepancy in the average value of air density from both formulas. The result of air density from CIPM 1981/91 calculation is 1.1963 kgm^{-3}, 0.0001 kgm^{-3} smaller than the result from CIPM 2007. Very small difference makes it insignificant to the uncertainty budget, but for an updated knowledge, it is better to use CIPM 2007 formula. Air buoyancy correction for both formulas is around 0.031 kgm^{-3}, and contributes as the third highest in the uncertainty budget for weight calibration. It makes quite an impact for the uncertainty analysis in the standard weight calibration.

ACKNOWLEDGEMENT

The authors would like to thank RCM LIPI and all of the staffs, especially from the mass metrology division, for all the help and knowledge in the making of this paper.

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